

Weyl metrics and wormholes

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Abstract

© 2017 IOP Publishing Ltd and Sissa Medialab srl. We study solutions obtained via applying dualities and complexifications to the vacuum Weyl metrics generated by massive rods and by point masses. Rescaling them and extending to complex parameter values yields axially symmetric vacuum solutions containing singularities along circles that can be viewed as singular matter sources. These solutions have wormhole topology with several asymptotic regions interconnected by throats and their sources can be viewed as thin rings of negative tension encircling the throats. For a particular value of the ring tension the geometry becomes exactly flat although the topology remains non-trivial, so that the rings literally produce holes in flat space. To create a single ring wormhole of one metre radius one needs a negative energy equivalent to the mass of Jupiter. Further duality transformations dress the rings with the scalar field, either conventional or phantom. This gives rise to large classes of static, axially symmetric solutions, presumably including all previously known solutions for a gravity-coupled massless scalar field, as for example the spherically symmetric Bronnikov-Ellis wormholes with phantom scalar. The multi-wormholes contain infinite struts everywhere at the symmetry axes, apart from solutions with locally flat geometry.

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Keywords

gravity, Wormholes

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